

# Transient Beam Loading Effects in Gas-Filled RF Cavities for a Muon Collider\*

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## Abstract

A gas-filled RF cavity can be an effective solution for the development of a compact muon ionization cooling channel. One possible problem expected in this type of cavity is the dissipation of significant RF power through the beam-induced plasmas accumulated inside the cavity (plasma loading). In addition, for the higher muon beam intensity, the effects of the beam itself on the cavity accelerating mode are non-negligible (beam loading). These beam-cavity interactions induce a transient phase which may be very harmful to the beam quality [1]. In this study, we estimate the transient voltage in a gas-filled RF cavity with both the plasma and conventional beam loading and discuss their compensation methods.

## INTRODUCTION

Conventional RF cavities are operated in a good vacuum condition to minimize RF breakdowns and residual gas scattering. For muon ionization cooling, an RF cavity filled with high-pressure hydrogen gas has been proposed. The high-pressure gas in the cavity functions as an energy absorber for the ionization cooling and also mitigates the RF breakdown problem associated with operation in a strong magnetic field. Unlike the vacuum cavity, the gas-filled cavity is subject to loading effects of two different sources: the beam-induced image charges in the cavity surface (beam loading), and the free electrons and ions generated inside the cavity (plasma loading) [2]. The plasma loading is represented as an additional resistive load while the beam loading is modeled as an additional current generator. This is consistent with our physical insight that even when the RF generator is off, the beam excites fields inside the cavity [3]. Also we note that the quality factor of the cavity can be decreased by the plasma loading but not by the beam loading.

For muon collider applications, the muon beams are formatted with only a dozen bunches in a pulse (e.g., 12 bunches in a 60 ns pulse at a 15 Hz repetition rate). Therefore, the beam loading effect does not reach a steady state, and the RF voltage seen by the last bunch will be considerably different from what the first bunch sees [4, 5]. In addition, for the gas-filled cavity, the plasma loading will reduce the quality factor simultaneously. In this paper, we investigate the transient response of the cavity under the influence of both the beam and plasma loadings.

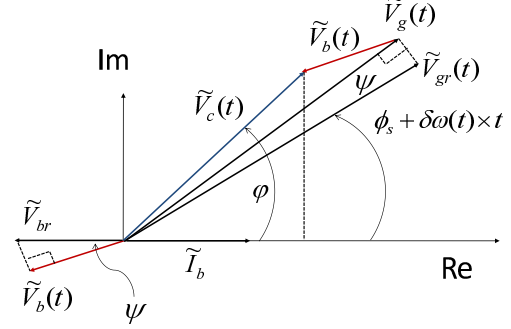


Figure 1: Phasor diagram showing  $\tilde{V}_c(t) = \tilde{V}_g(t) + \tilde{V}_b(t)$  for  $\gamma \ll 1$ . Due to a sudden turn-on of the beam-induced voltage, both amplitude and phase of the cavity voltage experience a transient behavior.

## EQUIVALENT CIRCUIT

First, let's assume voltages and currents to be varying at roughly the driving frequency  $\omega$  and express them in terms of phasors as  $V_c = \text{Re} [\tilde{V}_c(t)e^{j\omega t}]$ . In the slowly-varying approximation,  $|d\tilde{V}/dt| \ll |\omega\tilde{V}|$ , we get the following circuit equation [2]:

$$\frac{d\tilde{V}_c}{d\tau} + (1 - j \tan \psi + \gamma)\tilde{V}_c = \frac{2\beta_c}{1 + \beta_c}\tilde{V}_F - \frac{1}{2}Q_L \left[ \frac{R}{Q} \right] \tilde{I}_b, \quad (1)$$

where  $\tilde{V}_c = \tilde{V}_g + \tilde{V}_b$  is the cavity voltage phasor which is also the sum of the forward ( $\tilde{V}_F$ ) and reverse ( $\tilde{V}_R$ ) voltages. Here,  $\tau = t/T_f$  is time measured in units of filling time  $T_f = 2Q_L/\omega_0$ , and  $\gamma = Q_L(P_{ei}/\omega_0 U)$  is a damping coefficient due to the plasma loading. Further,  $Q_L$  is the loaded quality factor,  $U$  is the total stored electromagnetic energy, and  $P_{ei}$  is the average power absorbed by the plasma. The difference in driving frequency  $\omega$  from resonant frequency  $\omega_0$  is characterized by tuning angle  $\psi$ , which is given by  $\tan \psi = Q_L(\omega_0/\omega - \omega/\omega_0) = -2Q_L\delta\omega/\omega_0$ . For  $\delta\omega > 0$ , indeed  $\psi$  becomes negative.

## BEAM LOADING

The average DC current of a bunched beam of charge  $Q_b$  is approximated by  $I_{DC} = Q_b/T_b$ . Here, the bunch spacing  $T_b$  is assumed to be a sub-harmonic of the resonance RF frequency  $\omega_0$  of the cavity (i.e.,  $T_b = 2\pi/\omega_b = h \times 2\pi/\omega_0$

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with  $h$  a positive integer). The number of particles in a single bunch is  $Q_b/e$ . When beam bunches pass repetitively through the RF cavity, the effective voltage is the sum of the voltage supplied by the generator current and the wakefields of all the bunches [6].

We consider the case of a point-like bunch with the bunch length much shorter than the bunch spacing. Then, from the well-known mathematical identity of the delta function we have

$$\begin{aligned} I_b(t) &= Q_b \sum_{m=-\infty}^{\infty} \delta(t - mT_b - \Delta t) \\ &= \frac{2Q_b}{T_b} \text{Re} \left[ \frac{1}{2} + \sum_{n=1}^{\infty} e^{jn\omega_b(t-\Delta t)} \right]. \end{aligned} \quad (2)$$

Here, we consider a non-zero synchronous phase  $\phi_s = h\omega_b\Delta t$  with respect to the crest. In typical linacs, for example, to obtain simultaneous acceleration and synchrotron motion we need  $-\pi/2 \leq \phi_s \leq 0$ . In other words, the beam arrives earlier than the RF crest [3]. On the other hand, in a cooling channel with high dispersive elements, we set  $\phi_s > 0$  because the beam is above transition.

We assume the bunch spacing  $T_b$  is short compared to the cavity filling time  $T_f = 2Q_L/\omega_0$ , i.e.,  $T_b \ll T_f$ . In this case the beam-induced wakefields can be assumed approximately sinusoidal [1, 7]. Therefore, we restrict our discussion to the interaction of the beam with the fundamental cavity resonance mode. In other words, the beam current is represented by its component at the driving RF frequency. The interactions with higher order cavity mode can be minimized by dedicated damping antenna installed into the cavity [7]. When  $Q_L$  is large enough ( $T_b/T_f \approx \pi h/Q_L \ll 1$ ), only the term  $n = h$  in Eq. (2) falls within the bandwidth of the resonance. The terms with  $n \neq h$  are smaller than the  $n = h$  term by  $O(\pi h/Q_L)$  and can be neglected [8]. Now the beam current is given by the single tone signal in phasor notation as follows:

$$I_b(t) \approx \text{Re} \left[ \tilde{I}_b(t) e^{j\omega t} \right] \times H(t), \quad (3)$$

with  $\tilde{I}_b(t) = 2I_{DC} e^{-j\phi} e^{-j\delta\omega(t)t}$  and  $H(t)$  is the Heaviside step function. We consider the phase slippage between the beam repetition rate and the RF driving frequency, that is to say, the beam repetition rate is no longer a subharmonic of the driving frequency [4]. Note that, in general,  $\delta\omega(t)$  is a function of time. Before the beam injection  $t < 0$ , there is no phase slippage. When  $t \geq 0$ ,  $\delta\omega(t)$  varies slowly on the time scale of the cavity voltage change as we detune  $\omega$ .

We note that  $[R/Q]$  measures the efficiency of acceleration per unit stored energy at a given frequency and is defined by  $[R/Q] = |\tilde{V}_c|^2/(\omega_0 U)$ , where the amplitude of the cavity voltage is  $|\tilde{V}_c| = E_0 L_c T$ . Here, we use an average axial electric field amplitude  $E_0 = V_0/L_c$  with axial RF voltage  $V_0 = \int_{-L_c/2}^{L_c/2} E(r=0, z) dz$ . The transit time factor  $T$  is the ratio of the energy gained in the time-varying RF field to that in a DC field [3]. Equation (1)

Parameter	Symbol	Value	Unit
Muon momentum	$p_\mu$	200	MeV/c
Muon kinetic energy	$T_\mu$	120.6	MeV
Muon velocity	$\beta = v/c$	0.88	-
Transit time factor	$T$	0.855	-
RF frequency	$f$	805	MHz
Cavity radius	$R_w$	0.1425	m
Cavity length	$L_c$	0.1	m
Field gradient	$E_0$	20	MV/m
Coupling coeff.	$\beta_c$	1	-
Number of bunches	$N_b$	12	-
Total number of muons	$N_\mu$	$1 \times 10^{13}$	-
Muons per bunch	$Q_b/e = N_\mu/N_b$	$8.3 \times 10^{11}$	-
Average DC current	$I_{DC} = Q_b/T_b$	26.7	A
Cavity quality factor	$Q_w$	25411	-
Stored energy	$U$	3.05	J
Wall power dissipation	$P_w$	0.6	MW
Synchronous phase	$\phi_s$	64	degrees
R over Q	$[R/Q]$	189.8	Ohms
Cavity filling time	$T_f$	5.024	$\mu\text{s}$

Table 1: Example parameters for 805 MHz pill-box RF cavity with total number of muons of  $10^{13}$ .

can be normalized by the forward voltage amplitude  $\tilde{V}_F = E_{0,\max} L_c T$ . In the steady state with  $\beta_c \rightarrow 1$ ,  $\tan \psi \rightarrow 0$ , and  $I_b \rightarrow 0$ , we have  $\tilde{V}_c = \tilde{V}_F = \tilde{V}_g = E_{0,\max} L_c T$ .

## PLASMA LOADING

The average electron-ion (e-i) production rate during  $T_b$  is expressed by

$$\dot{N} = \frac{I_{DC}}{e} L_c \frac{\rho dE/dx}{W_i}. \quad (4)$$

When electrons are removed quickly through the attachment process to a electronegative gas (0.1%  $\text{O}_2$  for our case),

$$N_e(t) \approx \dot{N} T_b \exp[-(t - mT_b)/\tau], \quad (5)$$

for  $mT_b \leq t < (m+1)T_b$  with  $m = 0, 1, \dots$ , and

$$N_+ \approx N_- \approx N_i(t) = \sqrt{\dot{N} V / \beta_{ii}} \tanh \left( t \sqrt{\dot{N} \beta_{ii} / V} \right), \quad (6)$$

where  $\beta_{ii}$  is the ion-ion recombination rate (typically  $10^{-8} - 10^{-7} \text{ cm}^3/\text{s}$  at very high pressure) and  $V$  is the volume of the plasma column. Although the ions are accumulated more or less like a step-function, Eq. (6) is a good approximation on the average.

The average power dissipation is  $P_{ei} = P_i + P_e$ :

$$P_i \approx N_i(t) \times dw_i \times f, \quad (7)$$

with  $dw_i = e(\mu_i^+ + \mu_i^-) E_0^2 / 2f$  and  $\mu_i^+$  ( $\mu_i^-$ ) is the mobility of positive (negative) ions which is independent of the electric field for our parameter range. Also, there is finite electron contribution.

$$P_e \approx (\dot{N} T_b) \times dw_e \times f, \quad (8)$$

where  $dw_e = (1/T_b f) e E_0 \int_0^{T_b} e^{-t/\tau} v_d(t) |\cos(\omega t + \phi_s)| dt$ . The  $dw_e$  depends on time constant of the attachment  $\tau$  and the synchronous phase  $\phi_s$ . The electron drift

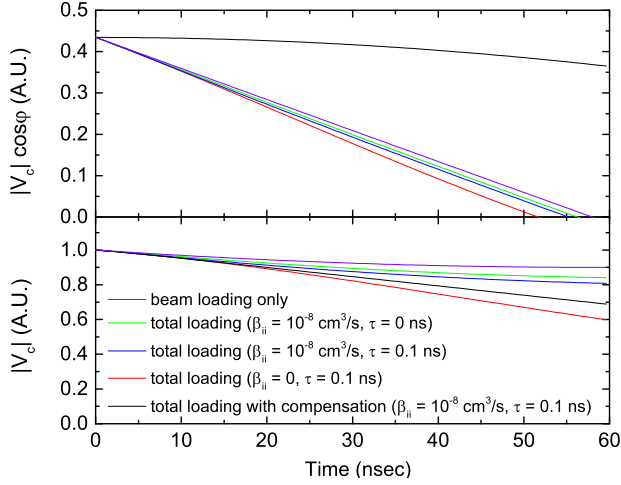


Figure 2: Effects of the beam and plasma loadings for several different physics scenarios.

velocity  $v_d(t)$  depends on the instantaneous value of the electric field. Note that electrons are not accumulated over the beam pulse because  $\tau \ll T_b$ , so the main contribution to the plasma loading comes from the ions.

## RESULTS

We solve the Eq. (1) numerically for the normalized cavity voltage  $\hat{V} = \hat{V}_c / (E_{0,\max} L_c T)$  in the coordinate system where the beam current aligns with the real axis. First, we investigate the case with no phase slippage [ $\delta\omega(t) = 0$ ]. Figure 2 indicates that the loading effects are dominated by the beam itself because of the significant change in the synchronous phase. The accelerating voltage  $|\hat{V}_c| \cos \varphi$  becomes zero at the end of the pulse.

The reduction in the cavity voltage per bunch can be estimated from the power loss as well, i.e.,  $\Delta|\hat{V}_c|/|\hat{V}_c| \sim P_b T_b / U$ , where  $P_b = I_{DC} |\hat{V}_c| \cos \phi_s$  is the power delivered to the beam. The induced voltage due to the image charge build-up is  $Q_b / C \sim 2\Delta|\hat{V}_c|$ .

The power loss for the beam loading is constant while the power loss from the plasma loading increases according to the ion accumulation. At higher ion densities, the ion-ion recombination will eventually saturate the ion accumulation. As the characteristic time  $\tau$  for the process  $e^- + O_2 + H_2 \rightarrow O_2^- + H_2$  is finite, there can be some power losses due to electrons as well.

## COMPENSATION METHOD

The variation in  $\text{Re}(\hat{V}) = |\hat{V}| \cos \varphi$  will cause an energy spread  $\Delta E / E \approx [\text{Re}(\hat{V}_{\text{last}}) - \text{Re}(\hat{V}_{\text{first}})] / \text{Re}(\hat{V}_{\text{first}})$  between the first and last bunches. To keep  $|\hat{V}| \cos \varphi$  constant, we allow slow frequency detuning  $\delta\omega(t)$ , in which we assume  $\delta\omega' T_b \ll \delta\omega \ll \omega_0$ . Suppose that  $\delta\omega$  introduces a small correction  $\delta\hat{V}$  to the original cavity voltage

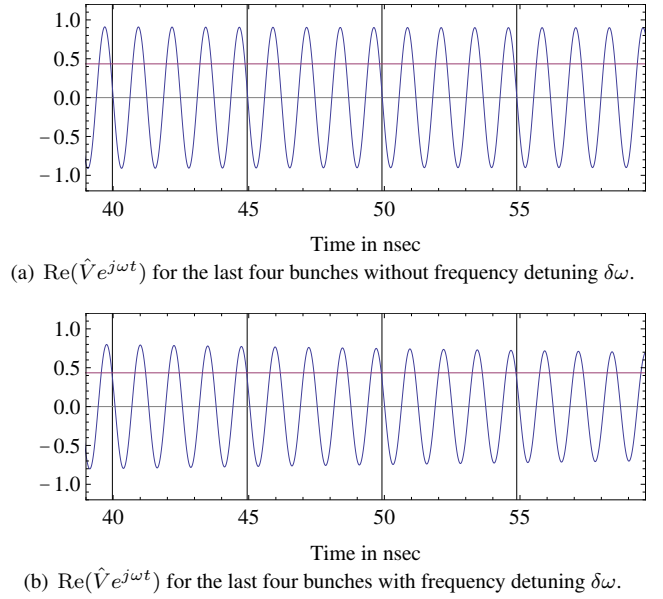


Figure 3: Effects of the frequency detuning  $\delta\omega$ . The black vertical lines indicates the timing of the bunch arrivals and the red horizontal lines are the target accelerating voltage.

$\hat{V}_0$  in such a way that  $\text{Re}[\hat{V}_0(t) + \delta\hat{V}(t)] = \text{Re}[\hat{V}_0(0)]$ . The required detuning is

$$\delta\omega(t) = \frac{1}{\text{Im}(\hat{V}_0)} \left[ \frac{d\text{Re}(\delta\hat{V})}{dt} + (1 + \gamma) \frac{\text{Re}(\delta\hat{V})}{T_f} \right]. \quad (9)$$

Figure 3 demonstrates the effect of the frequency detuning. To keep  $|\hat{V}| \cos \varphi$  constant, we increase  $\cos \varphi$  gradually over the beam pulse. In this case, the bucket area becomes inevitably smaller [6]. Compensation by increasing forward RF power is not practical as the voltage drop is too rapid and filling time is too slow.

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